

# APPLICATION NOTE

## Operational Modal Analysis of a Wind Turbine Wing using Acoustical Excitation

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*Operational Modal Analysis, also called Ambient Modal or Output-only Modal, is a technique where modal parameters are estimated from response data without knowing the input loading force. The method has, for over a decade, been used for parameter estimation on civil structures such as bridges and towers where artificial excitation and determination of forces present a problem.*

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## Introduction

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Classical, mobility-based modal analysis uses techniques which relate a known vibration response to a known force excitation using estimators such as H1 and H2. Such estimators carry information on the system under test, are independent of the input loading, and even work in cases where noise affects the input or output signal.

When the force is unknown, some assumptions have to be made. If the structure was excited by a white-noise force signal, the response would carry the information of the system under test. However, white noise never occurs naturally and the model must take the (unknown) excitation source into account. In operational modal analysis, we have to work with the so-called 'combined' model, consisting of the following parts:

- the system under test (our final result)
- frequency-dependent, random-noise excitation
- computational noise
- measurement noise
- harmonics from rotating parts

The purpose of the operational modal analysis estimators is first to fit the combined model to the experimental data containing all these components, and thereafter to extract the system parameters leaving the undesired components behind. Operational modal analysis includes a range of estimators divided into two main groups: Frequency Domain Decomposition and Stochastic Subspace Identification.

The Frequency Domain Decomposition (FDD) technique used for operational modal analysis is an extension of the classical frequency-domain approach. The classical approach is based on signal processing using the Fourier Transform and the modal transformation. The modal transformation in its output-only formulation is seen in (1). The modal information is extracted by singular-value decomposition for each frequency line in the FFT. Information on the number of modes is held in the number of singular values, and the mode shapes in the singular vectors (see Ref. [1] and Ref.[7]).

$$G_{yy}(j\omega) = \sum_{k=1}^N \frac{d_k \Phi_k \Phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\Phi}_k \bar{\Phi}_k^T}{j\omega - \bar{\lambda}_k} \quad (1)$$

Here,  $G_{yy}$  is the auto- and cross-power spectral densities of the response signals,  $y$ , for white-noise input and limited modal coupling,  $\omega$  is the angular frequency,  $\lambda_k$  is the

pole location (i.e., resonance frequency and damping) for the  $k$ th mode, and  $\phi_k$  is the mode shape for the  $k$ th mode.

Stochastic Subspace Identification (SSI) is, in contrast to the FDD technique, a method that uses time-domain techniques. SSI is formulated and solved using state-space formulations of the form:

$$x_{t+1} = Ax_t + w_t$$

$$y_t = Cx_t + v_t$$

where  $x_t$  represents the Kalman sequences that are found in SSI by a so-called orthogonal-projection technique (Ref. [3]).

It can be shown (Ref. [2]) that by performing a modal decomposition of the A-matrix and introducing a new state vector, the equation becomes:

$$z_{t+1} = [\mu_j]z_t + \Psi e_t$$

$$y_t = \Phi z_t + e_t$$

where  $\mu_j$  is a diagonal matrix holding the discrete poles, i.e., resonance frequency and damping for the modes, and where the matrix  $\Phi$  is holding the left-hand mode shapes (physical mode shapes) and the matrix  $\Psi$  is holding the right-hand mode shapes (non-physical mode shapes). The right hand mode shapes are also referred to as the initial modal amplitudes (Ref. [4]).

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## Measurement Setup

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**Fig. 1**  
*Wind-turbine wing with acoustic excitation. Observe loudspeaker beneath the wing*



The test object for this investigation is a 1:5 scale wind-turbine wing. It is a detailed model of one of the blades from a 675 kW wind turbine. The wing was made for lab investigations of static as well as dynamic parameters. Fig. 1 shows a picture of the measurement setup. The wing itself is supported by a console which is regarded as stiff compared to the wing itself.

24 accelerometers were mounted in two rows along the wing. Two time-recordings were taken, one with the accelerometers perpendicular to the surface (Z-direction) and the other with them pointing in the direction of rotation (X-direction). To determine the combined modes in this two-dimensional model, the results from the two recordings were linked together; the wing is considered as stiff in the direction of its length so vibrations in this direction were disregarded.

The aim of the measurements was to determine the lower modes for flutter investigations.

Modes in the audible frequency range, typically 30–400 Hz were also of interest due to potential noise problems.

The wing was exposed to an acoustic load by means of a loudspeaker placed underneath the wing. Although the aim was to use operational modal analysis for the test, the sound level was also measured and analysed.

Fig. 1 shows a microphone placed in front of the wing. This measurement was performed to ensure the presence of energy in the frequency range of interest. However, the spectral distribution does not need to be flat.

To compare the results with classical input/output-based modal analysis, mobility measurements, using hammer excitation with subsequent curve-fitting in modal analysis software, were performed.

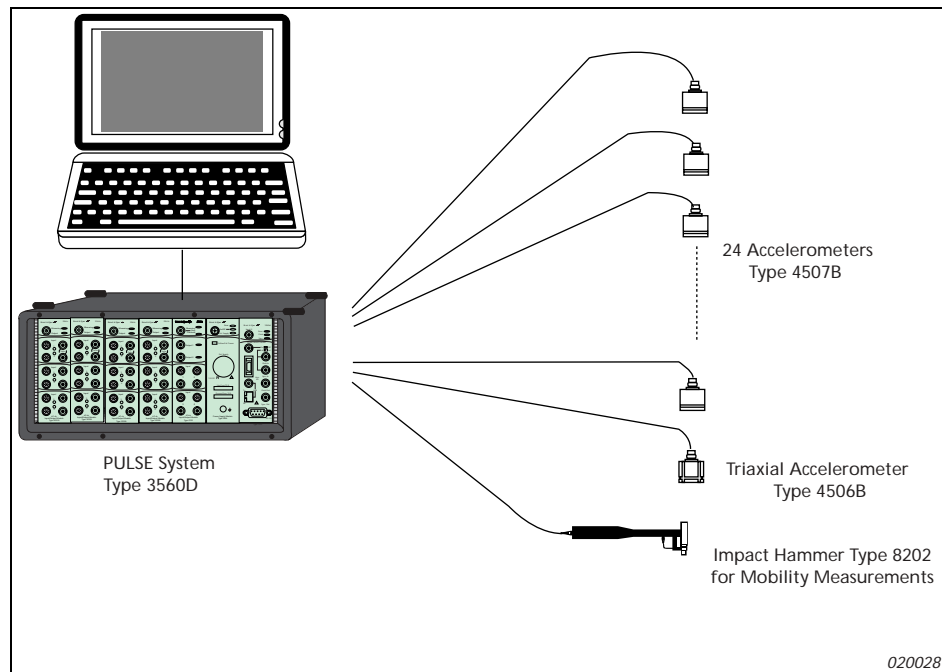
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## Measurement and Data-processing Equipment

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The measurements for the operational modal analysis, as well as the mobility-based modal analysis, were performed using the Brüel & Kjær PULSE™ Multi-analyzer System as shown in Fig. 2.

*Fig. 2 29-channel PULSE™ system used for data acquisition and analysis*



Modal Test Consultant™, a dedicated application software package, was used for the measurements and data validation. Modal Test Consultant™ supports the generation/import of geometry, definition of measurement sequences on the various points and directions (DOFs), and subsequent transfer/export of DOF-labelled measurement data.

For operational modal analysis, Modal Test Consultant™ provides for the acquisition of the response data as well as various FFT-based validation tools prior to processing in the Operational Modal Analysis (OMA) software (Ref [8]). For classical modal analysis, Modal Test Consultant™ provides the mobility measurements (Frequency Response Functions) and validation in terms of the coherence function, for example. The OMA software takes the raw time data and, based on this, estimates the modal parameters.

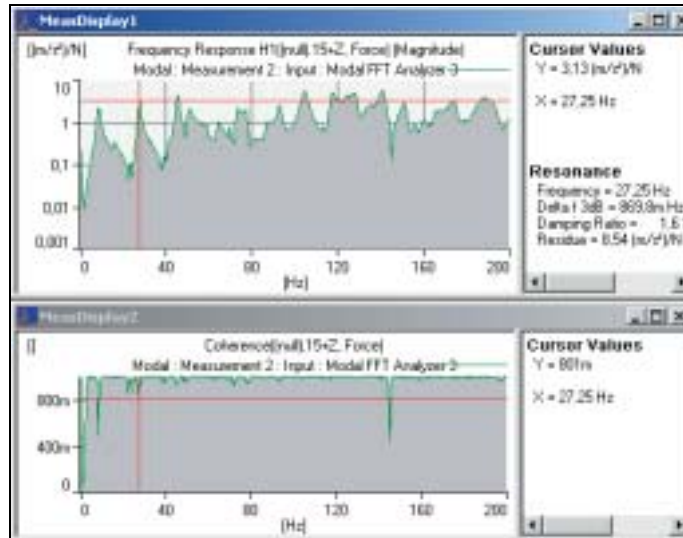
The program uses two different techniques – a non-parametric technique based on Frequency Domain Decomposition (FDD) and a parametric technique working on raw data in the time domain, a data-driven Stochastic Subspace Identification (SSI) algorithm.

The program provides different validation tools such that the results derived from the different techniques can be compared. ME'scopeVES™ was used to extract the modal model from the mobility measurements.

## Results

### Mobility Measurements

*Fig. 3*  
Example of  
frequency response  
function  
(accelerance)  
and  
corresponding  
coherence function



Mobility measurements were performed using impact-hammer excitation. Random impact was used since the frequency span of 200 Hz with 800 FFT lines gives a record length of 4 s. The structure was excited at one of the 'free-end' corner points and oriented at an angle of approximately 45 degrees to both the X- and the Z-axes to ensure excitation of all modes. The response was measured in the X- and Z-direction of the 24 points. The 48 frequency-response functions were measured in two measure-

ments using 25 channels (one force signal and 24 response signals). Fig. 3 shows one of the frequency-response functions (accelerance) and the corresponding coherence function. The frequency-response functions were curve-fitted and the modal parameters extracted for comparison with the modal model extracted using operational modal analysis.

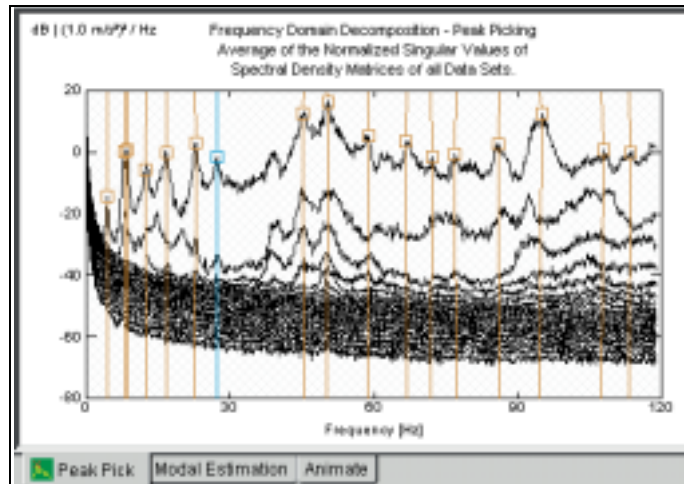
### Operational Modal Analysis

Acoustic excitation from a loudspeaker situated below the structure was used for the operational modal analysis. The acoustic signal was a low-frequency, random signal.

60s of time samples in a frequency span up to 200 Hz (sampling frequency of 512 Hz) were captured with measurements made at 24 points in both X- and Z-directions. This was done in two sessions (data sets) by first measuring the Z-direction and then the X-direction. Two directions from a triaxial accelerometer mounted close to one of the free-end corner points provided the reference signals used to link the results from the two data sets together.

Both the FDD and SSI methods were used in the extraction of the modal parameters. A number of peaks appear very clearly in the FDD, especially in the low-frequency range below 100 Hz as seen in Fig. 4.

**Fig. 4**  
*Frequency Domain Decomposition in terms of the average of normalised, singular values of spectral-density matrices of all data sets*



It is assumed that these peaks have been caused by structural resonances and not by a response to a high-level force excitation in a narrow band. 17 modes were detected below 110 Hz (see Fig. 4). The frequency and damping values were determined using a SDOF model applied in a user-definable frequency band around the peak.

A number of analyses were performed using the SSI method. A maximum state-

space dimension of 200, corresponding to a maximum of 100 modes, is applied in a frequency span up to 200 Hz. With the Canonical Variate Analysis (CVA) algorithm, models with a state-space dimension of around 140 produce models which agree with the measured data for frequencies down to approximately 40 Hz. This is verified by looking at the stabilisation diagram and synthesising the response auto- and cross-spectra from the model and comparing this with the measured spectra. Higher model order does not give much better agreement.

**Fig. 5**  
*Stabilisation diagram for the CVA algorithm. + (red), × (green) and × (yellow) indicate stable, unstable and noise modes respectively*

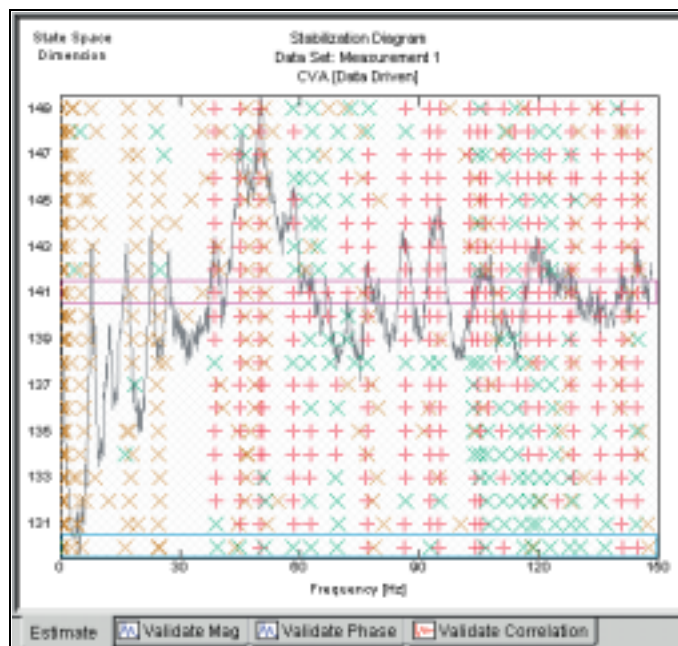
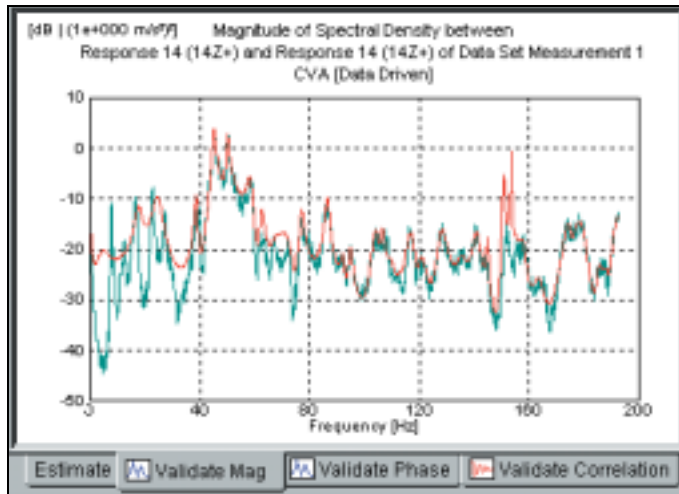


Fig. 5 shows a stabilisation diagram using the CVA algorithm. A mode whose modal parameters (frequency, damping and MAC) change within user-definable intervals, compared to the previous lower model order, is called a stable mode and is indicated by a red-coloured +. If one of the parameters changes more than is defined by these intervals, the mode is defined as unstable, indicated by a green ×. Modes with damping values outside a user-definable range, e.g., > 5% or negative, are called noise modes and indicated by a yellow-coloured ×.

Fig. 6 shows a comparison of the synthesised response autospectrum, for a selected state-space dimension of 141, with the measured response autospectrum. Good agreement is found in the frequency range 40 Hz to 200 Hz, with the exception of the two modes around 150 Hz. In the low frequency range below 40 Hz, however, the model is not very good and use of higher state-space dimensions does not improve it very much. Therefore, low-pass filtering and decimation to 50 Hz is performed on the time data and new SSI analyses are performed. Fig. 7 shows a comparison between a synthesised autospectrum, using a state-space dimension of 123 from the new SSI analysis, and the measured autospectrum.

**Fig. 6**  
 Example of a comparison of a synthesised response autospectrum with the measurement. Frequency span is 200 Hz and selected state-space dimension is 141

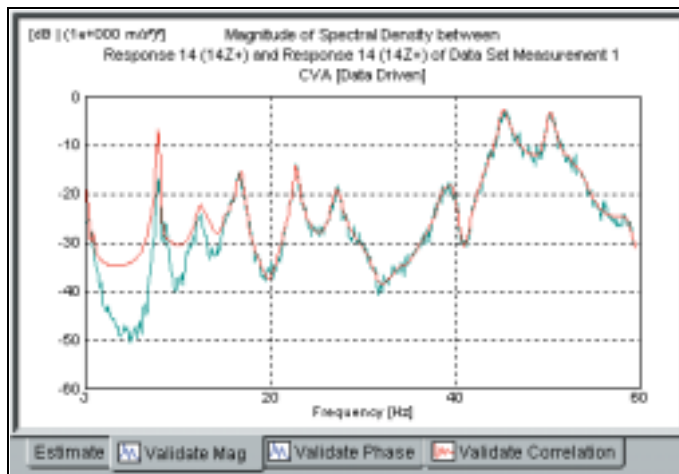


Here, very good agreement is found down to approximately 10 Hz. The low-frequency modes, including those below 10 Hz, are very well estimated from the FDD method and further low-pass filtering and decimation for the SSI methods were not performed.

The other two SSI algorithms provided by the OMA software, the Principal Component and Unweighted Principal Component, were applied as well. They gave

similar results, thus verifying the model. The frequencies of the estimated modes up to 110 Hz from the different methods, including mobility measurements, Enhanced FDD and CVA, are compared in Table 1.

**Fig. 7**  
 Example of comparison between a synthesised autospectrum and the measured autospectrum. A CVA model on the 50 Hz low-pass filtered and decimated signals with a state space dimension of 123 is used

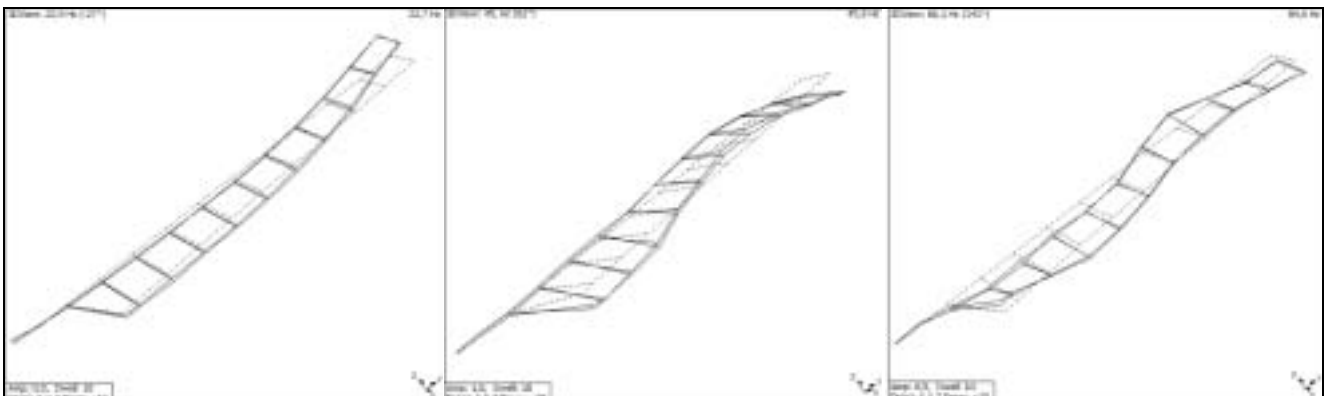


The FDD method provides a very good estimation of the modes, especially in the lower frequency range, and it is the only method which finds the mode at 4.4 Hz. There is good agreement with the modes estimated (curve-fitted) from the mobility measurements.

The CVA method, by combining the results from the two frequency spans, also provides good agreement with the modes estimated from the mobility measurements.

The different mode shapes are compared by overlay of the animated wire frames and by calculation of the cross-MAC functions. As examples, the estimated shapes of the modes at 22 Hz, 45 Hz and 85 Hz from the mobility measurement and the CVA method are compared in Fig. 8. Good agreement is seen and the cross-MACs are calculated to be 0.78, 0.95 and 0.82 respectively.

**Fig. 8** Comparison of the estimated shapes for the modes at 22 Hz, 45 Hz and 85 Hz from the mobility measurement and the CVA method



**Table 1**  
*Comparison of the modal frequencies estimated from:*  
*a) curve fitting on mobility functions ('classical' modal analysis)*  
*b) FDD method*  
*c) CVA method in frequency span of 200 Hz*  
*d) CVA method in frequency span of 50 Hz*

	a) Modes from Mobility Test	b) OMA FDD Enhanced	c) OMA CVA 200 Hz	d) OMA CVA 50 Hz Decimated
Mode	Frequency (Hz)	Frequency (Hz)	Frequency (Hz)	Frequency (Hz)
1		4.41		
2	8.16	7.83		7.79
3	8.84	8.55		
4		12.5		12.2
5	15.4	16.6		16.9
6	22.9	22.7		22.7
7	27.3	27.1		27.1
8	38.6	39.1	39.0	38.6
9				39.6
10	45.0	45.4	45.3	45.1
11	50.1	50.2	50.1	
12	59.0	58.6	58.6	
13	63.6		62.5	
14	69.2	66.8	66.3	
15	72.9	71.7		
16	77.2	77.0	76.9	
17	86.1	86.2	86.5	
18	92.8			
19	96.4	94.5	94.5	
20	104		104	
21	107	107	106	

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## Conclusions

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### Results

Operational modal analysis is shown to be an efficient tool for the determination of modal parameters when the input loading force, for some reason, is unknown or impossible to measure. When the results from the mobility test are compared with those from the FDD method, there is a good correlation, even at higher frequencies where the peaks are smeared out and the modes are relatively close.

The time-domain methods require long time-records depending on the lowest frequency of interest. The length of the time record required for the SSI technique depends upon the time period of the lowest modal frequency of interest. The factor between this period time and the required record length depends upon the number of modes and modal coupling.

Comparing the results from the mobility test with the SSI technique, it is seen that the correlation is good at modes above 39 Hz (column c), Table 1). The direct way to get the modes at the lower frequencies as well would be, according to the theory, to make longer time recordings and use higher state-space dimensions, thereby extending both the measurement and the calculation times. Instead, a low-pass filtering and decimation process on the same data is performed and the modes at the lower frequencies are identified as seen in column d) in Table 1. The estimated lower mode shows good correlation with those estimated from the mobility test.

It has been shown that operational modal analysis provides an efficient tool for determining the modal parameters of the model of a wind-turbine wing during simulated operational conditions (see Fig. 1). Previously, such tests have been performed using torsional shakers inserted into the drive shaft of the turbine, which is both a time-consuming, expensive and, to some extent, risky method. The OMA technique offers new possibilities for the determination of modal parameters from response data only.

## References

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- [2] Brincker, R. and Andersen, P.: "ARMA Models in Modal Space", Proc. of the 17th International Modal Analysis Conference, Kissimee, Florida, 1999.
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- [5] Brincker, R., Andersen, P. and Møller, N.: "Output-only Modal Testing of a Car Body subject to Engine Excitation", Proc. of the 18th International Modal Analysis Conference, San Antonio, Texas, February 7 – 10, 2000.
- [6] Møller, N., Brincker, R. and Andersen, P.: "Modal Extraction on a Diesel Engine in Operation", Proc. of the 18th International Modal Analysis Conference, San Antonio, Texas, February 7 – 10, 2000
- [7] Brincker, R., Zhang, L. and Andersen, P.: "Output-only Modal Analysis by Frequency Domain Decomposition", ISMA 25.
- [8] Møller, N., Brincker, R., Herlufsen, H., Andersen, P.: "Modal Testing of Mechanical Structures subject to Operational Excitation Forces, IMAC XIX.

## Product Literature

### *Product Data*

- Modal Test Consultant Type 7753 (Literature Number BP 1850)
- Operational Modal Analysis Type 7760 (BP 1889)

### *System Data for PULSE™ Type 3560*

- Hardware (BU 0228)
- Software (BU 0229)

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